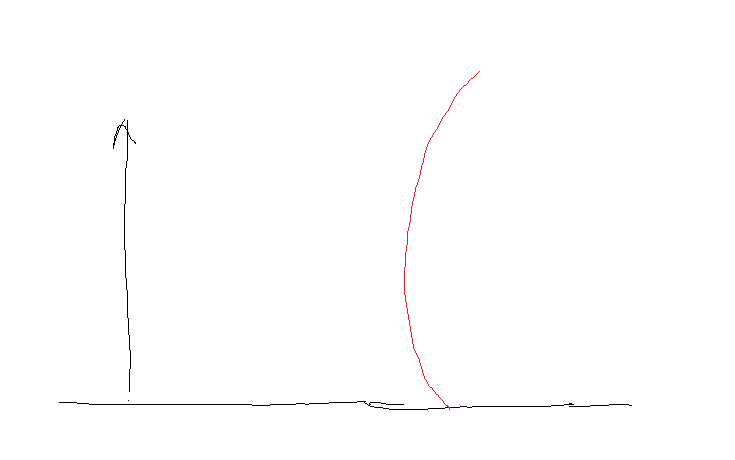
**Mirrors**

**Example**

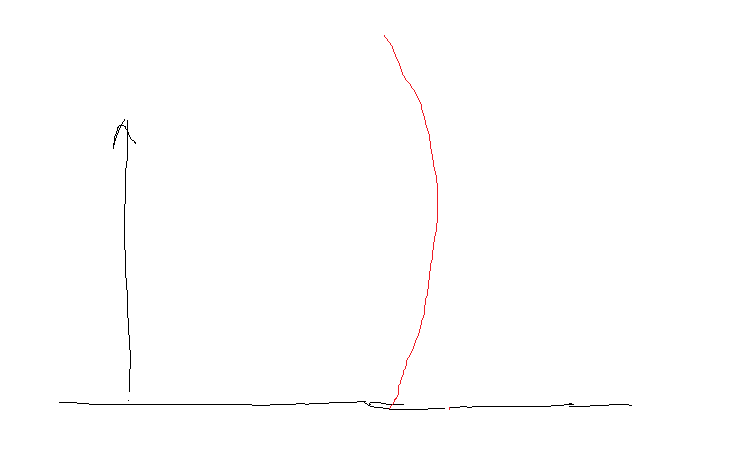
Suppose have curvy mirror certain distance away. How short would you appear and how wide? How could you make someone appear wider.



Just use equation. And then could make them appear wide by using a larger focal length in the dimension you want to appear bigger, like a plane mirror.

**Example**

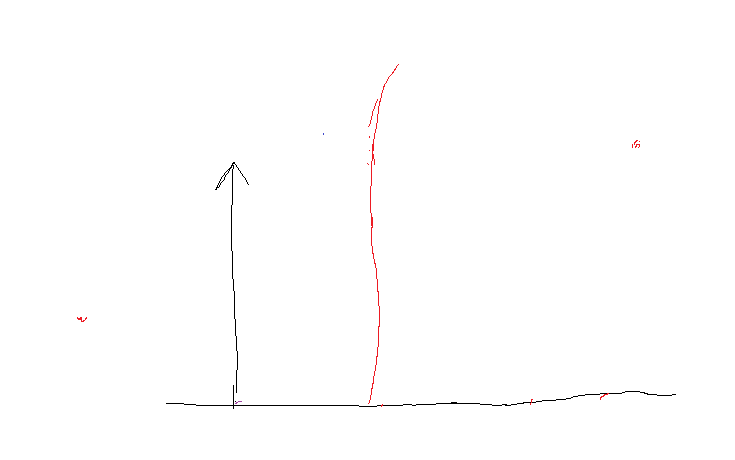
Suppose have curvy mirror certain distance away. How short would you appear and how wide? How could you make someone appear skinnier.



Just use equation. And then could make them appear wide by using a larger focal length in the dimension you want to appear bigger, like a plane mirror.

**Example**

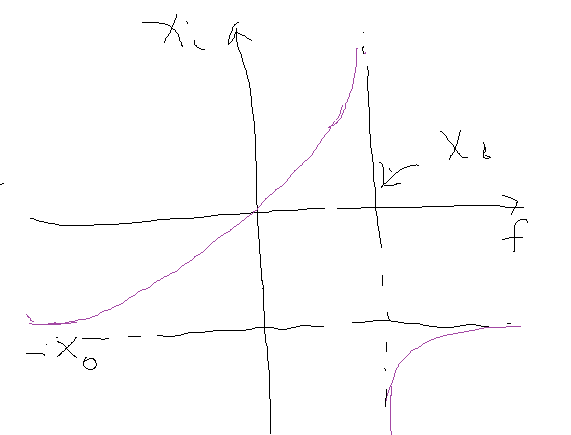
Suppose have funhouse mirror with given focal lengths and distance. What approximately would you see?





and can do opposite curvy kind.





General plot of xi vs. f looks like this. So as start with R = 0 concave then xi is at 0 since focal length is 0. And then as open it up image will go all the way to infinity until focal length surpasses xo in which case image will be projected at -∞ and then as f continues to increase the image will approach the mirror till it gets to - xo. And then as start to bend the mirror the other way so that it becomes convex, the image distance is still same and then it starts to approach closer and closer to the mirror to where it will be equal to zero when R again gets to 0 from the other side.

But image will always be further away than object when have virtual image from concave mirror. Man, had to think about that one.

**Example**

Suppose you are a certain distance from a mirror and you want to create a giant image of your head (m = 5) and a certain distance away, say d = 10m. Then what type and focal length of mirror do you need and where must you stand?



and,



so solve.

5. A concave shaving mirror has a radius of curvature of 32 cm. It is positioned so that the (upright) image of a man's face is 2.4 times the size of the face. How far is the mirror from the face?

Using the lens equation:



and magnification formula:



So filling this into the lens equation:



15. If you would like to take an object 10cm tall, and produce an upright image 3m away from the mirror and 5cm tall, what kind of mirror do you need? What will be its focal length.



Also, the mirror is diverging (convex) because the image is upright and smaller. So q = -3m. So,



Solving for p, we get p = 6. Then plugging into the mirror equation:



So *f* = -6.

**Problem 4.**

If you would like to take an object 10cm tall, and produce an upright image 3cm away from the mirror and 5cm tall, what kind of mirror do you need? What will be its focal length?

You need a convex mirror since only those mirrors are capable of producing upright images smaller than the object itself. Consequently, the image distance will be negative. Using the mirror equation:



and magnification formula:



So filling this into the lens equation:



**Example**

You have a wizard-of-oz head that’s 40cm tall. Want to project it 6m away and be 2m tall. Where should you place lens and what should its focal length be?

M = -2/0.4 = -5 🡪 -(6-x)/x = -5 🡪 6-x = 5x 🡪 x = 1m. And then 1/x + 1/xi = 1/f 🡪 1/1 + 1/(6-1) = 1/f 🡪 solve for f.

**Example**

If you have a concave mirror, with focal length of 10cm, where should you place an object so that the image will be magnified by a factor of 5 and be upright?

From the lens equation…



From the magnification requirement…



Now solve for q here and plub it into the lens equation…





So



3. If you have a concave mirror, with focal length of 10cm, where should you place an object so that the image will be magnified by a factor of 5 and be upright? Specify the distance from the mirror.

From the lens equation…



From the magnification requirement…



Now solve for q here and plug it into the lens equation…





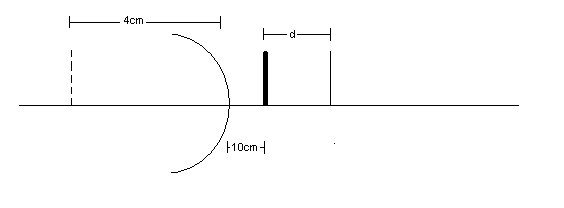
So



**Example**

Suppose you’re standing at an ATM with one of those convex mirrors they place on them sometimes so you can see behind you. Suppose you are standing 25cm away from the mirror and that it has an approximate radius of curvature of 10cm. If, looking at the mirror, you see someone who appears to be 4cm behind the mirror, how far away from you are they?

The (not to scale exactly) drawing is shown below. You are the bold line, and the person is the skinny line (and the person’s image is the dotted line).



We wish to find d. We can use the lens equation. The object distance is p = 10 + d. This is the distance the object is from the mirror, obviously. The image distance is q = -4cm. the negative sign is necessary because the image is behind the mirror. And the focal length is f = -R/2 = -5cm. The negative sign is due to the fact that the mirror diverges light rays that impinge upon it. So…



So the person would be 10cm away from you.

9. Dennis the dentist uses a curved mirror to veiw the back side of a tooth. What kind of mirror, and with what focal length, should the dentist use to create a virtual image of the tooth with magnification m = 2.4, when it is 1.2cm behind the tooth. Draw a ray diagram illustrating the mirror, the object, and rays forming the image.

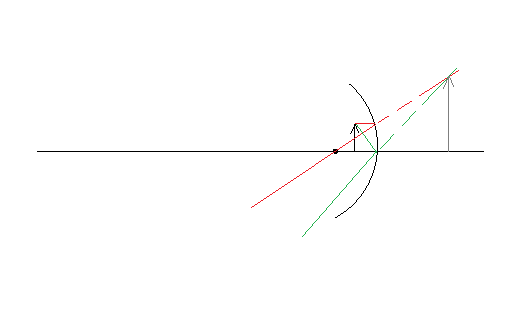


and,



plugging this into the first equation, we get f:





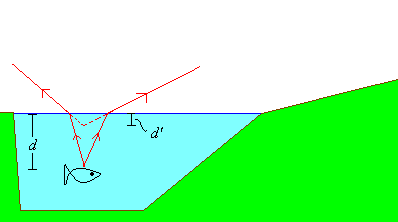
**ATM alternate**

Same kind of problem as ATM. Just say that you are standing a distance xo in front of mirror and your image height appears to be so many cm. (a) what is the mirrors focal length and how far behind the mirror is your image? Then say that you notice in mirror some dude who appears to be appear to be so tall in the mirror and so far away (further) from mirror. What is their actual height and distance?

**Refracting Surfaces**

**Example 1**

How does actual depth of a fish compare to its apparent depth?



The light rays reflecting off of the fish will refract further away from the normal once they hit the water air interface – from Snell’s law. If we trace the rays back to their apparent origin, this will give us the apparent depth of the fish. We can determine d′ from Snell’s law, but we’ll just use the equation above,



So if the apparent depth is, say, 1m, then the actual depth is:



What is the magnification?



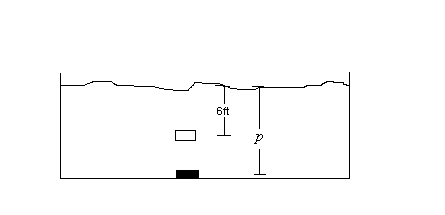
So the image isn’t magnified any. So while the fish does appear closer than actual (d′ < d), it doesn’t appear magnified any.

**Example**

Do fish example for real with Snell’s law.

2. Suppose you see a the drain at the bottom of the deep end of a swimming pool. If it appears to be 6ft below the surface of the water, how deep is it really? Remember that for water n = 1.33.

**.** We’re looking for p. q, the image distance, is *negative* 6ft. since the image appears on the same side as the object.



The equation we use is,



n1 is for water, and n2 would be that of air, since the light would be going from the object (in the water) into the air. The radius of curvature of the surface is ∞ since the interface between the air and water is flat. So we have,

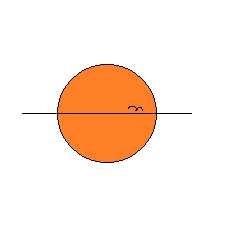


So,



So the pool is actually 8 ft deep.

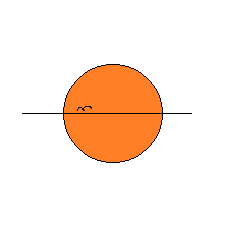
**Question 10**. A mosquito is trapped in a spherical blob of amber of radius r = 5cm. It appears to be about 8cm away from the left edge. How far from the edge is it actually? Amber’s index of refraction is 1.54. Note your eye is on the left.



We have:



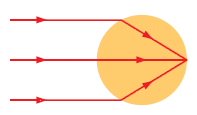
**Question 10**. A mosquito is trapped in a spherical blob of amber of radius r = 5cm. It appears to be about 3cm away from the inside edge. How far from the edge is it actually? Amber’s index of refraction is 1.54.



We have:



6. In the figure below, a beam of parallel light rays from a laser is incident on a solid transparent sphere of index of refraction *n*. If a point image is produced at the back of the sphere, what is the index of refraction of the sphere? Assume the index of refraction of the outside medium is 1.3.



‘Lens’ equation is given by:



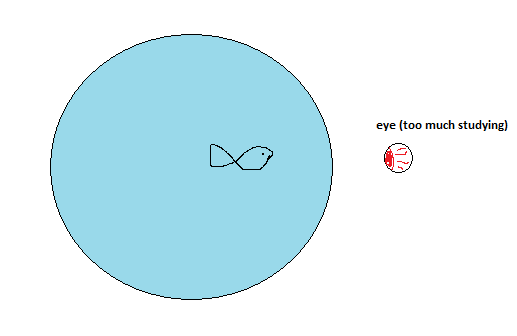
For parallel light rays, the object distance would be considered infinite. And q would be 2R. Filling these in:



**Question 6**. A goldfish lives in a 50-cm-diameter spherical fish bowl. The fish sees a cat watching it. If the cat's face is 20 cm from the edge of the bowl, how far from the edge does the fish see it as being? (You can ignore the thin glass wall of the bowl; and index of refraction of water is 1.33).



10. A goldfish called silver lives in a 50cm diameter spherical fish bowl. If it appears to be 10cm away from the edge, with height 3cm, how far from the edge is it really? And what is its real height? Index of refraction of water is n = 1.33.



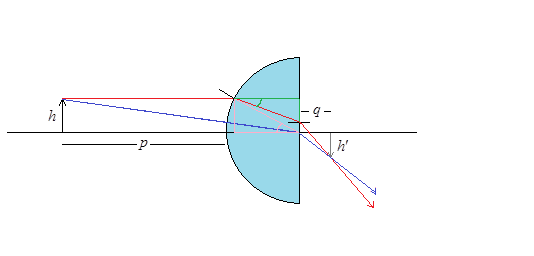


and its height is:



**Problem 5.**

In class we considered how refraction can form images via lenses. In our derivation of the lens equation, namely 1/p + 1/q = (n-1)[1/r1 – 1/r2], we implicitly used an approximation that the lens be thin. Let’s discard this approximation for a simple case. Suppose you place an h = 10cm tall object a distance p = 85cm from the front end of a semi-circle lens. If the index of refraction of the lens is n = 2.5, and its radius is r = 40cm, where (q) will the image of the object form? What will be its height (h´)? You can solve this problem by considering the path of two light rays as shown. One starts horizontally and gets refracted twice. The other starts perpendicular to the circular surface and therefore only gets refracted at the flat surface. We did similar problems in class you might recall. OK, and then finally, use the lens equation to calculate q and h´ and see how close they are to the exact answers you just laboriously calculated.



Let’s consider the red ray first. Look at pink triangle. The pink angle at its base, which happens to also equal the angle of incidence, is θ = sin-1(10cm/40cm) = 14.5◦. And from Snell’s law, the angle of refraction will therefore be:



The angle that the refracted ray makes w/r to the horizontal (the green angle) is 14.5◦ - 5.75◦ = 8.75◦. This is also the angle of incidence at the second interface. And the height at which this ray will hit that interface is, using the green triangle y = 10 – Rcos(14.5◦)tan(8.75◦) = 4. The angle at which it emerges is, from Snell’s law:



So the equation for the emerging ray will be:



Now consider the blue ray. The angle of incidence at the second interface is

θ = tan-1[10cm/(85+40)] = 4.57◦. And so by Snell’s law, the refracted angle will be:



and so the equation of this line will be:



These rays will intersect at when,



This is q. And the height of the image will be the value of y at the intersection. This is:



According to the lens equation, we have:



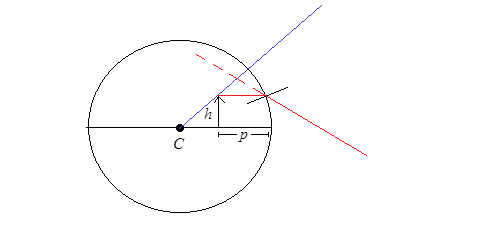
This is quite a bit off. However, if we interpret this number to be the distance between the image and the *center* of the lens, then it is pretty close to our answer above. And the new height would be:



As you can see, these values are a bit off. And this is due to the fact that the width of our lens (40cm) is comparable in size to p (85cm); thus we cannot treat it as a thin lens with negligible width.

**Problem 7.**

This is not a poke-ball. This is an object inside a spherical refracting medium (n = 2.4) surrounded by air (n = 1). The radius of the medium is R = 40cm. The height of the object is 10cm, and it is a distance p = 12cm from the surface. How far away from the surface is the virtual image located, and what is its height? I recommend drawing two rays – one going from object tip radially away from C, and another going from object horizontally initially, with then gets refracted. Compare to the answers predicted by the refracting surfaces equation.



Setting the origin at the center of curvature, the equation for the blue line is:



For the other we need the angle of incidence. This is θ = sin-1(h/R) = sin-1(5/40) = 7.18◦. Therefore the angle of refraction is: (2.4)sin(7.18) = 1sinθ → θ = sin-1(2.4sin(7.18)) = 17.5◦. We need the angle it makes with respect to the horizontal. This is θ = 17.5 – 7.18 = 10.3◦. So then the equation for the refracted ray is:



Setting these two equal we get:



which is q = C – x = 17.4 cm away from the surface. And the height of the image is:



Comparing to the refracting surface equation we get:



and the new height would be:

10. Suppose you’re operating a movie projector. As such, a picture on the film reel (which is your object) is placed 10cm from the lens, and the lens is to project the image onto a screen 15m away. What focal length must the lens have? A





**Question 11.**  A goldfish lives in a 75cm diameter spherical fish bowl. The fish sees a cat watching it. If the cat’s face is 20cm from the edge of the bowl, how far from the edge does the fish see it as being? You can ignore the width of the bowl’s glass wall. And note the index of refraction of water is 1.33.



So the fish will see it 32cm away.

**Lenses**

7. If a person’s near point is 0.8m, what focal length lens does he need to correct his farsightedness, i.e., to change his effective near point to 25cm? You may assume the glasses are right next to the eye so that there is no space between them.

Using lens equation:



11. A person wears glasses with focal length f = 30cm. Are they nearsighted or farsighted? What is the their nearpoint or farpoint?

Because the lens is convex, they are farsighted. Their farpoint is governed by:

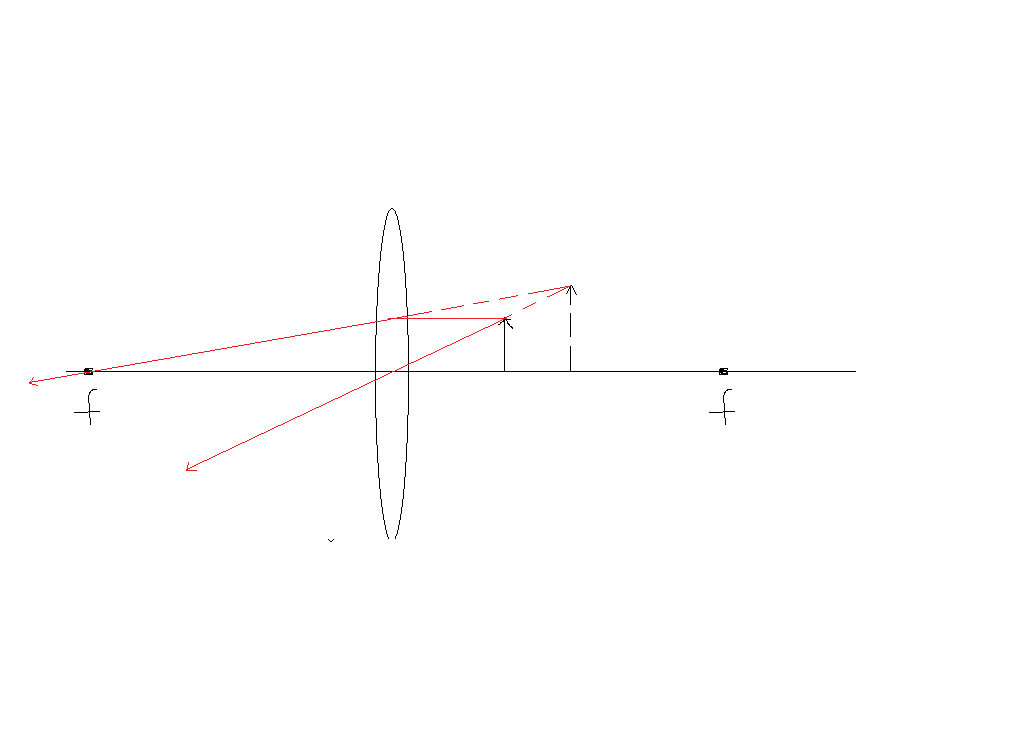


**Question 11**. If someone is farsighted, with a near point of xnp. = 40cm, what power lens should you prescribe to enable him to see clearly at x = 25cm? In addition to doing this mathematically, also use ray tracing techniques to locate the object and image in a diagram.

We need lens to project an image of object at 25cm to 40cm. So,



which equates to a power of P = 1/f = 1/0.667 = 1.5 diopters. And diagram looks like,



**Example 1**

Suppose the near point of a patient’s eye is 50cm. What focal length must a corrective lens have to enable the eye to see clearly an object 25cm away if the distance between the glasses and eye is 2cm?

That the person has a near point of 50cm indicates that he is farsighted. Anyway, the purpose of the glasses is to form an image of the object at the near point of the eye so that the eye can focus it. So we need the lens to take an object that is 25cm from the eye (23cm from the glasses) and focus its image at a distance of 50cm from the eye (48cm from the glasses). So using our lens equation…



**Example 2**

A particular nearsighted person can’t see objects clearly when they are beyond 1.5m (the far point). What focal length should the prescribed contact lens have to correct this problem?

So the far point of the person is 1.5m. So we need to take objects at infinity and focus them at q = -1.5m. So we have,



So the near sighted person needs a diverging lens as expected. What is the new near point, assuming it was, say 10cm before? The new near point is the closest an object can be so that he can still see it. So now we use the lens equation and determine where an object has to be for its image to be focused at -10cm.



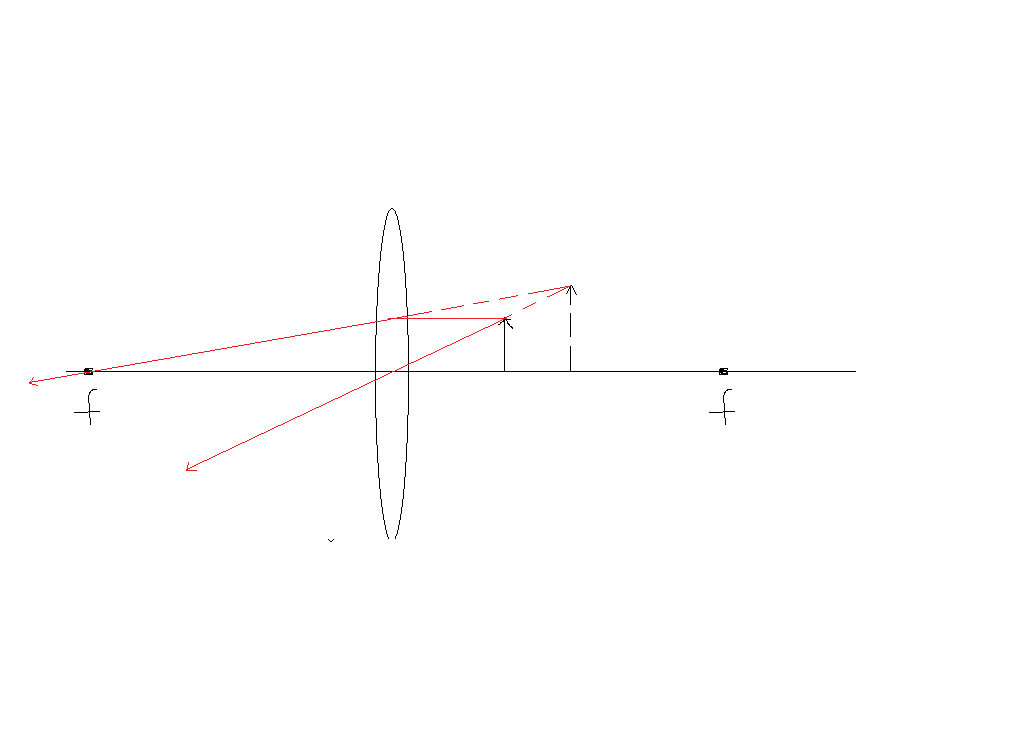
So the new near point is at 10.71 – farther away than before as expected, but not by much.

**Question 11**. Gandalf is farsighted, both metaphorically, and actually. His near point is xnp. = 40cm. What lens power does Gandalf prescribe himself so that he can see clearly at x = 25cm? In addition to doing this mathematically, use ray tracing techniques to locate the object and image in a diagram.

We need lens to project an image of object at 25cm to 40cm. So,

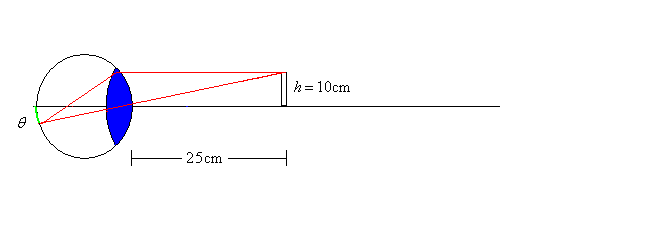


and diagram looks like,



**Example 1**

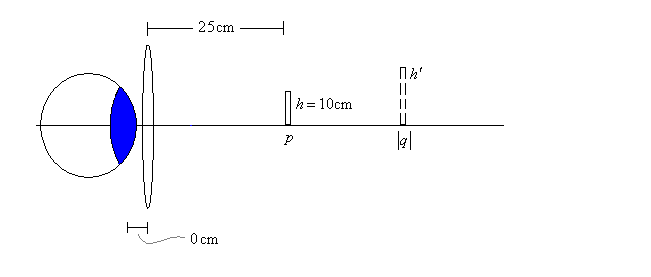
Suppose we put an object (10cm tall) at your near point then – say 25cm. And suppose that we have a lens f = 50cm, positioned 0cm from your eye – like a contact lens. We want to first determine the angular size of the object w/o the lens, and then the angular size of the object w/ the lens. So without first…



(the dimensions are exaggerated a bit for clarity). So the angular size is:



Now if we place the lens in front of the eye, 0cm away, an image of the object will be projected at q which is:



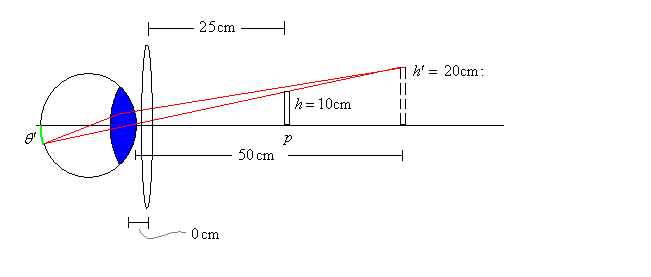
at,



and the height is:



And so the image will be projected 50 + 0 = 50 cm away with height 20cm. But what size will this appear as to the eye?



So your eye will see the image at that location, and with apparent size,

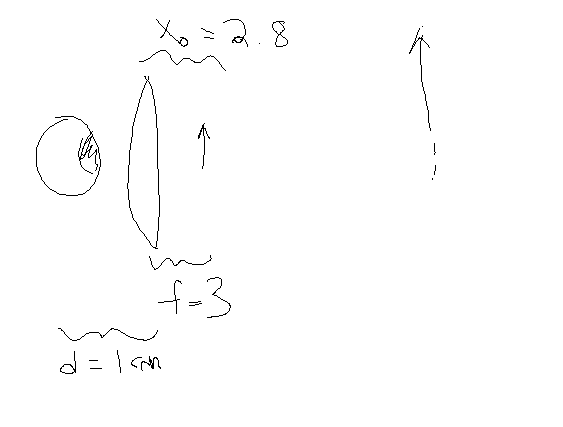


So there is no magnification.



**Example**

Do example of magnifier like above but with reference to your near point. As shown below, xi = f2/δ - f, so make sure that xi > 25cm. So for instance in the problem below we have xi = 32/0.2 - 3 = 42. So we’re good.



So then,



and then magnification is mh = 42/2.8 = 15. And so angular magnification is:

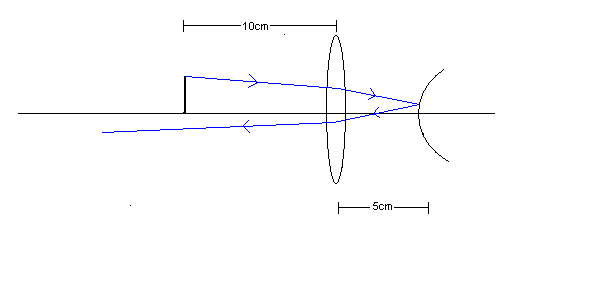


**Combinations**

**Example**

Suppose you have an object 10cm away from a convex lens (focal length 7cm), which itself is 5cm away from a convex mirror (radius of curvature 5cm). Locate the image you would see if you were standing in front of the lens (on the same side of the lens as the object). Ascertain whether it is real or imaginary, and its magnification.

If you are standing on the same side of the lens as the object, then the image you see will come from the light that goes through the lens, reflects off of the mirror, and travels back through the lens – to your eyes. Pictured below is the path of such a typical light ray



We shall proceed in 3 steps. We’ll find the image produced by the lens when the light goes through it the first time. Then we’ll see what image the mirror produces when it reflects this light, and then finally what image the lens produces with the reflected light.

1. ***First image created by lens***,

We proceed with the lens equation,



So the image that would be produced by the lens (were the mirror not there) is 23.3cm beyond the lens, which places it 18.3cm beyond the mirror. The magnification of the image is,



2. ***Image created by the mirror***

We treat this first image formed by the lens as the object for the mirror. Since the first image landed 18.3cm beyond the mirror, the object distance is considered to be *negative* 18.3cm. Also recall the focal length for the convex mirror is negative; . So we have,



and



Since q2 = -2.9, the image formed by the mirror is located 2.9 cm *behind* the the mirror.

1. ***Second image created by lens***,

The second image (created by the mirror) is 2.9cm behind the mirror, which is 7.9cm in front of the lens. So p3 = 7.9. Proceeding with the lens equation,



So the final image is 61.4cm past the lens. And the magnification is,



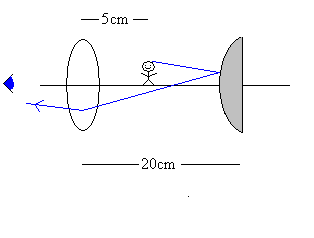
So the image you see would be 61.4cm to the left of the lens. It would be *real* since the final image distance is positive (61.4). And the overall magnification would be,



So the image would be 2.9 times larger than the object, and would be *inverted* since m is negative.

**Example**

Suppose you have a convex lens and concave mirror arranged as shown.



Let the focal length of the lens be 10cm, and that of the mirror, 5cm (positive or negative? you decide!). Consider the image formed by light reflecting off of the mirror and then going through the lens. How far away is it from the lens? And is it virtual or real?

First we consider the image formed by the mirror: Note that it has a negative focal length, since it diverges light rays. All convex mirrors have negative focal lengths.

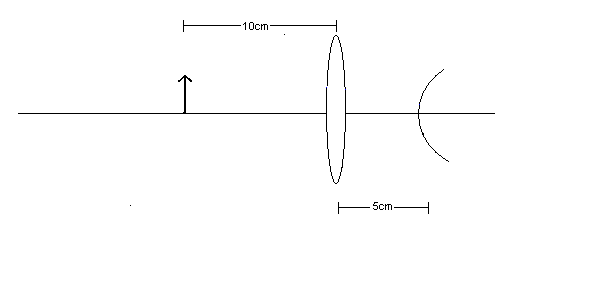


So the first image formed by the mirror is 3.75cm to the right of (behind) the mirror. Now we use the lens equation again. We treat the image just found as the object for the lens.

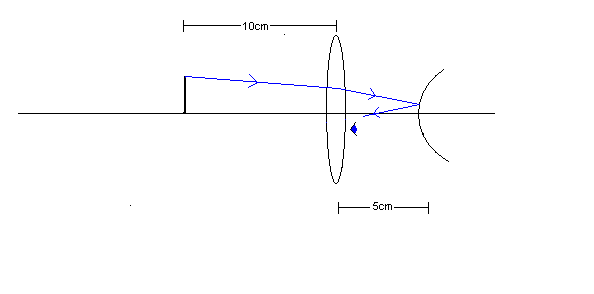


So the final image is 17.3 cm to the left of the lens. And since the image distance was positive, it must be a real image.

5. Suppose you have an object 10cm away from a convex lens (focal length 7cm), which itself is 5cm away from a convex mirror (radius of curvature 5cm). Locate the image you would see if you were standing between the lens and the mirror, looking towards the mirror. Ascertain whether it is real or imaginary.



If you are standing in between the lens and mirror, then the image you see will come from the light that goes through the lens, reflects off of the mirror. Pictured below is the path of such a typical light ray



We shall proceed in 2 steps. We’ll find the image produced by the lens when the light goes through it the first time. Then we’ll see what image the mirror produces when it reflects this light.

1. ***First image created by lens***,

We proceed with the lens equation,



So the image that would be produced by the lens (were the mirror not there) is 23.3cm beyond the lens, which places it 18.3cm beyond the mirror. The magnification of the image is,



2. ***Image created by the mirror***

We treat this first image formed by the lens as the object for the mirror. Since the first image landed 18.3cm beyond the mirror, the object distance is considered to be *negative* 18.3cm. Also recall the focal length for the convex mirror is negative; . So we have,





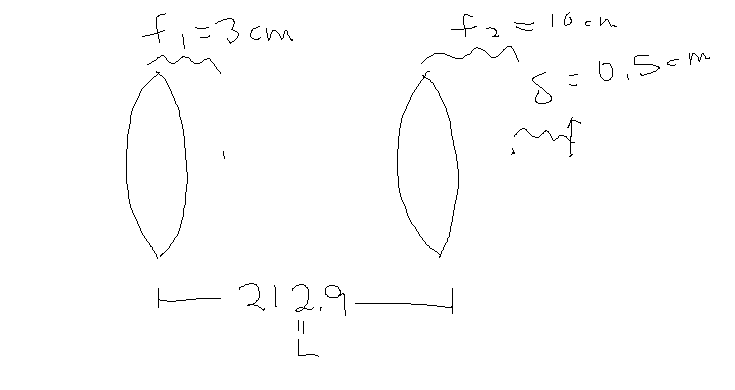
Since q2 = -2.9, the image formed by the mirror is located 2.9 cm *behind* the mirror. And since we have a negative image distance, the image is virtual.

**Example**

Do microscope example. As a quick guide, xi is generically given by:



And note that we want xi < L < xi + f1, similar to telescope example below. Or, more quickly, we need L just under f22/δ + f2 + f1, i.e., L < f1 + f2 + f22/δ. And so to construct problem in head, I would say, suppose we have two lenses f1 = 3cm, and f2 = 10cm. We place the object a distance δ = 0.5cm away from the focal point of the second lens (and hence xi ~ 10­2/0.5 + 10 = 210cm – big microscope – whatever) and the first lens a distance L such that 210 < L < 213 → L = 212.9cm perhaps Then what will be the angular magnification of the image?



Then for real we have:



which means the magnification will be m2 = 210/10.5 = 20. And then for the second image:

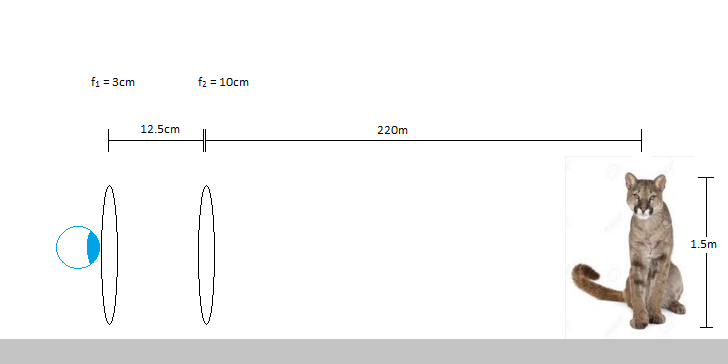


and so magnification would be: m2 = -87/2.9 = 30. So angular magnification is:



**Question 12.** You are walking down the street when you think you see a 1.5m tall puma. Just to be sure, you pop the lenses out of your glasses and construct a telescope. Situation looks like this. What is the angular magnification of the puma?

Note when constructed such a problem, you want the sum of the focal lengths to be just greater than the distance between the lenses, i.e., L < f1 + f2, so that the second image will be virtual.



The angular size of the puma, without lenses, is:



Now let’s see where and how large the image will be. So second (f2) lens will create image of puma at:



to the left. And the height of the image will be:



And then this will be the object for the second lens a distance 2.5cm away from it, which will create an image at:



with a height,



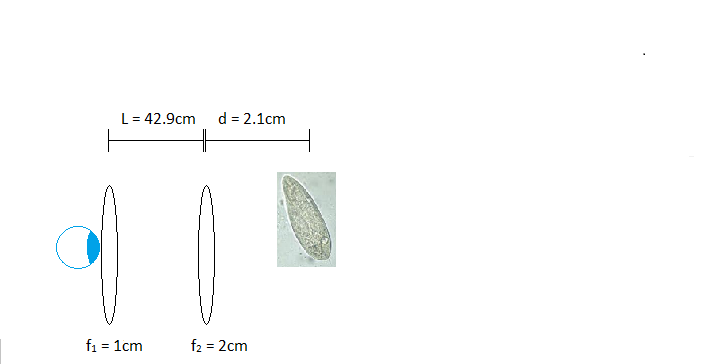
So the angular size of the image will be:



which is an angular magnification of:



12. Say hi to Pete the paramecium, sitting under a microscope. What angular magnification does the microscope impart to his sexy image? Assume a standard nearpoint of x = 25cm.



So, working out where the first image will be:



and the height of the image will be:



The position of the second image is:



and the height of the second image will be:



the angular size of the image will be:



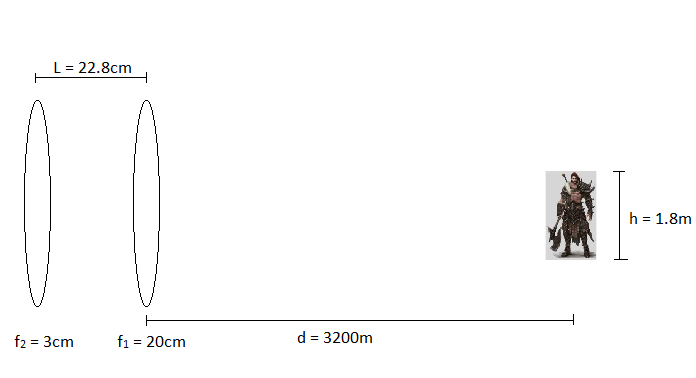
The angular size of the object, at the near point is:



So the magnification is:



**Question 12.** It’s 476 and you see the Barbarian hordes coming your way (sorry Germans). If a Barbian is 1.8m tall, and 3200m away, what angular magnification will your telescope give you? Compare this to the generic formula for the magnification.



So angular size of our object is:



Now we must figure out the size and position of the final image. Those of the first image will be:



and height is:



Now for the next lens. So our ‘object’ is 2.8cm from the second lens. Plugging this into the lens equation:



and the height



and so the angular size of the final image is:



and so the angular magnification is:



Generic formula predicts mθ = f1/f2 = 20/3 = 6.67. So pretty close.